SOLUTION OF THE EXERCISE FOR THE METHOD: TRANSMISSION LOAD RELIEF (TLR)

1) We set :

$C_{12} = C_{23} = 2C \qquad C_{31} = C \qquad$ W

$x_{12} = x_{23} = x/2 \qquad x_{31} = x \qquad$ p.u.

where :

$y_{12} = y_{23} = 2y \qquad y_{31} = y \qquad$ p.u.

In the linearized case, we have :

$$P_{ab} = \frac{U_a U_b}{X_{ab}} (\theta_a - \theta_b) \qquad \text{W}$$

In dimensionless magnitudes, it is :

$$p_{ab} = \frac{1}{x_{ab}} (\theta_a - \theta_b) \qquad \text{p.u.}$$

Taking into account as reference $\theta_3 = 0$, for the given network, we can write:

$$p_{12} = \frac{1}{x_{12}} (\theta_1 - \theta_2) = y_{12} (\theta_1 - \theta_2) = 2y (\theta_1 - \theta_2) \qquad \text{p.u.}$$

$$p_{23} = \frac{1}{x_{23}} \theta_2 = y_{23} \theta_2 = 2y \theta_2 \qquad \text{p.u.} \qquad (1)$$

$$p_{31} = \frac{1}{x_{31}} (-\theta_1) = -y_{31} \theta_1 = -y \theta_1 \qquad \text{p.u.}$$

They can be shown as a function of the injections; we have the following linearized system:

$\mathbf{P} = \mathbf{Y} \boldsymbol{\theta} \qquad \boldsymbol{\theta} = \mathbf{Y}^{-1} \mathbf{P}$

with : **P** = vector of injections
Y = matrix of nodal admittance
θ = vector of voltage angles of the nodes

where :

$$\begin{bmatrix} p_{g1} \\ p_{g2} \end{bmatrix} = \begin{bmatrix} y_{12} + y_{31} & -y_{12} \\ -y_{12} & y_{12} + y_{23} \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \qquad (2)$$

The system (2) becomes :

$$\begin{bmatrix} p_{g1} \\ p_{g2} \end{bmatrix} = \begin{bmatrix} 3y & -2y \\ -2y & 4y \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \qquad (3)$$

By inversing (3), we obtain :

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{1}{4y} \begin{bmatrix} 2 & 1 \\ 1 & 3/2 \end{bmatrix} \cdot \begin{bmatrix} p_{g1} \\ p_{g2} \end{bmatrix}$$

The developed form is :

$$\theta_1 = \frac{1}{2y} P_{g1} + \frac{1}{4y} P_{g2} \qquad \text{rad}$$

$$\theta_2 = \frac{1}{4y} P_{g1} + \frac{3}{8y} P_{g2} \qquad \text{rad} \qquad (4)$$

By combining the relations (1) and (4), we get :

$$p_{12} = 2y(\theta_1 - \theta_2) = \frac{1}{2} p_{g1} - \frac{1}{4} p_{g2} \qquad \text{p.u.}$$

$$p_{23} = 2y\theta_2 = \frac{1}{2} p_{g1} + \frac{3}{4} p_{g2} \qquad \text{p.u.}$$

$$p_{31} = -y\theta_1 = -\frac{1}{2} p_{g1} - \frac{1}{4} p_{g2} \qquad \text{p.u.}$$

In real magnitudes :

$$P_{12} = \frac{1}{2} P_{g1} - \frac{1}{4} P_{g2} \qquad \text{W}$$

$$P_{23} = \frac{1}{2} P_{g1} + \frac{3}{4} P_{g2} \qquad \text{W}$$

$$P_{31} = -\frac{1}{2} P_{g1} - \frac{1}{4} P_{g2} \qquad \text{W}$$

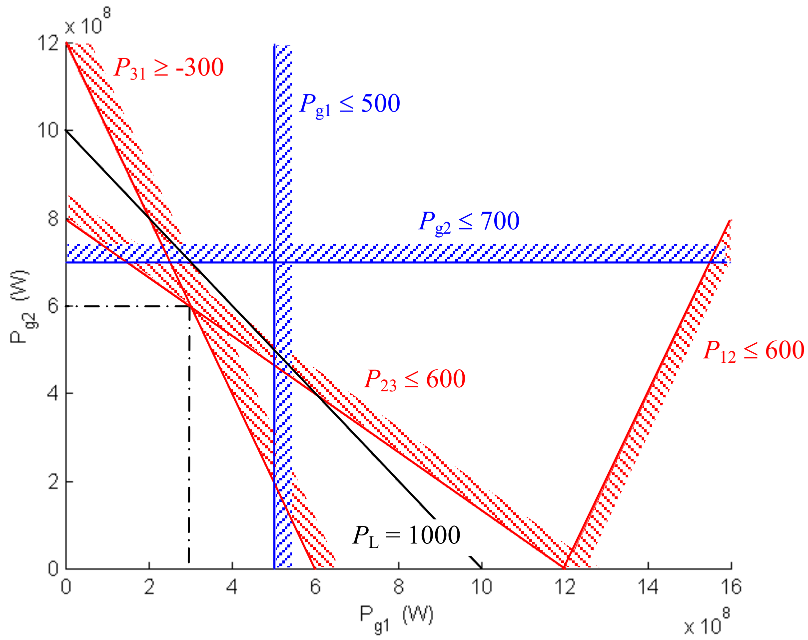
2) We have the following constraints:

$$P_L = 1000 \qquad \text{MW}$$
$$\Delta P_L = P_L - P_{g1} - P_{g2} \geq 0 \qquad \text{MW}$$

$$0 \leq P_{g1} \leq 500 \qquad \text{MW}$$
$$0 \leq P_{g2} \leq 700 \qquad \text{MW}$$

$$-600 \leq \frac{1}{2}P_{g1} - \frac{1}{4}P_{g2} \leq 600 \qquad \text{MW}$$
$$-600 \leq \frac{1}{2}P_{g1} + \frac{3}{4}P_{g2} \leq 600 \qquad \text{MW}$$
$$-300 \leq -\frac{1}{2}P_{g1} - \frac{1}{4}P_{g2} \leq 300 \qquad \text{MW}$$

Graphically, the constraints can be represented as follow (the hachures represent the part of the plan that the constraints are not respected) :



From the following figure, we obtain the production plan that minimizes the load shedding at bus 3 or maximizes Pg1 and Pg2:

$P_{g1} = 300$ MW and $P_{g2} = 600$ MW

For which we have :

$\Delta P_L = 1000 - 300 - 600 = 100$ MW